Solution :

(iii) Let $\sin^{-1}\frac{3}{5}=\alpha$ *and* $\sin^{-1}\frac{5}{13}=\beta$

$$\sin\alpha=\frac{3}{5} \quad and \quad \sin\beta=\frac{5}{13}$$

$$\cos\alpha\frac{4}{5} \quad and \quad \cos\beta=\frac{12}{13}$$

$$\cos\left\{\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{5}{13}\right\}=\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$=\frac{4}{5}\times\frac{12}{13}-\frac{3}{5}\times\frac{5}{13}=\frac{33}{65}$$

3. Show that $\sec^2(Tan^{-1}2)+\cos ec^2(\cot^{-1}2)=10$

Solution:

Let $Tan^{-1}2=\alpha$ *and* $\cot^{-1}2=\beta$

$$Tan\alpha=2 \quad and \quad \cot\beta=2$$

$$\text{LHS} = \sec^2\alpha+\cos ec^2\beta=1+Tan^2\alpha+1+\cot^2\beta=1+4+1+4=10$$

4. Show that (i) $Tan^{-1}\frac{1}{7}+Tan^{-1}\frac{1}{13}+Tan^{-1}\frac{2}{4}=0$ **(ii)** $Tan^{-1}\frac{1}{2}+Tan^{-1}\frac{1}{5}+Tan^{-1}\frac{1}{8}=\frac{\pi}{4}$

(iii) $Tan^{-1}\frac{3}{4}+Tan^{-1}\frac{3}{5}-Tan^{-1}\frac{8}{19}=\frac{\pi}{4}$

(iv) $Tan^{-1}\frac{1}{7}+Tan^{-1}\frac{1}{8}=\cot^{-1}\frac{201}{43}+\cot^{-1}(18)$

Solution :

(i) Let $Tan^{-1}\frac{1}{7}=\alpha$ $Tan^{-1}\frac{1}{13}=\beta$ $Tan^{-1}\frac{2}{9}=\gamma$

$$Tan\alpha=\frac{1}{7} \quad Tan\beta\frac{1}{13} \quad Tan\gamma=\frac{2}{9}$$

$$Tan(\alpha+\beta)=\frac{Tan\alpha+Tan\beta}{1-Tan\alpha Tan\beta}=\frac{\frac{1}{7}+\frac{1}{13}}{1-\frac{1}{91}}=\frac{\frac{20}{91}}{\frac{90}{91}}=\frac{2}{9}$$

$$Tan(\alpha+\beta-\gamma)=\frac{Tan(\alpha+\beta)-Tan\gamma}{1+Tan(\alpha+\beta)Tan\gamma}=\frac{\frac{2}{9}-\frac{2}{9}}{1+\frac{4}{8}}=0$$

$$\because \alpha+\beta-\gamma=0 \Rightarrow Tan^{-1}\frac{1}{7}+Tan^{-1}\frac{1}{13}-Tan^{-1}\frac{2}{9}=0$$

Solution :

(ii) $Tan^{-1}\frac{1}{2}+Tan^{-1}\frac{1}{5}+Tan^{-1}\left(\frac{1}{8}\right)$

Let $Tan^{-1}\frac{1}{2}=\alpha \quad Tan^{-1}\frac{1}{5}=\beta \quad Tan^{-1}\frac{1}{8}=\delta$

$$Tan\alpha=\frac{1}{2} \quad Tan\beta=\frac{1}{5} \quad Tan\gamma=\frac{1}{8}$$

$$Tan(\alpha+\beta)=\frac{\frac{1}{2}+\frac{1}{5}}{1-\frac{1}{10}}=\frac{7}{9}$$

$$Tan(\alpha+\beta+\gamma)=\frac{\frac{7}{9}+\frac{1}{8}}{1-\frac{7}{9}\times\frac{1}{8}}=\frac{56+9}{72}\times\frac{72}{72-7}=1$$

$$\alpha+\beta+\delta=\frac{\pi}{4}\Rightarrow Tan^{-1}\frac{1}{2}+Tan^{-1}\frac{1}{5}+Tan^{-1}\frac{1}{8}=\frac{\pi}{4}$$

Solution :

(iii) Let $Tan^{-1}\frac{3}{4}=\alpha \quad Tan^{-1}\frac{3}{5}=\beta \quad and \; Tan^{-1}\frac{8}{19}=\delta$

$$Tan\alpha=\frac{3}{4} \quad Tan\beta=\frac{3}{5} \quad and \; Tan\delta=\frac{8}{19}$$

$$Tan(\alpha+\beta)=\frac{\frac{3}{4}+\frac{3}{5}}{1-\frac{9}{20}}=27\Rightarrow Tan\left(\alpha+\beta-\delta\right)=\frac{Tan\left(\alpha+\beta\right)+Tan\delta}{1-Tan\left(\alpha+\beta\right)Tan\delta}$$

$$=\frac{\frac{27}{11}+\frac{8}{19}}{1+\frac{27}{11}\times\frac{8}{19}}=\frac{\frac{513-88}{209}}{\frac{209+216}{209}}=1$$